TRINITY COLLEGE

# Year 12 Mathematics Applications
# Test 3 2019

Section 1 Calculator Free

**Graph Theory**

**STUDENT'S NAME:** Solutions

**DATE**: Wednesday 8th May **TIME:** 30 minutes **MARKS**: 31

**INSTRUCTIONS:**

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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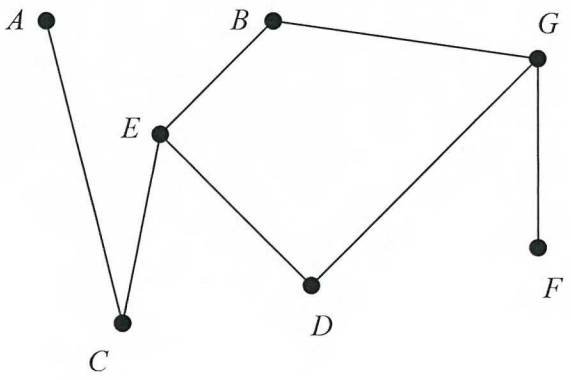
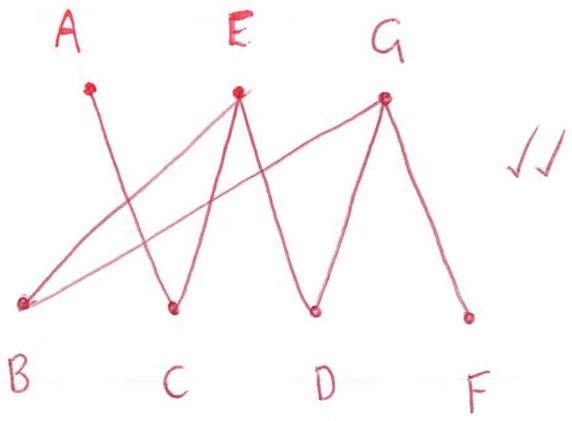
1. (3 marks)

Given that the following graph is bipartite, re-draw the graph as a more obvious bipartite graph and list the two groups of vertices.




✓✓

{A, E, G} ✓

{B, C, D, F}

2. (3 marks)

(a) A planar graph has 11 regions (faces) and 15 edges, how many vertices does it have? [2]

$V + F - E = 2$

$6 + 11 - 15 = 2$ ✓

$V = 6$ ✓

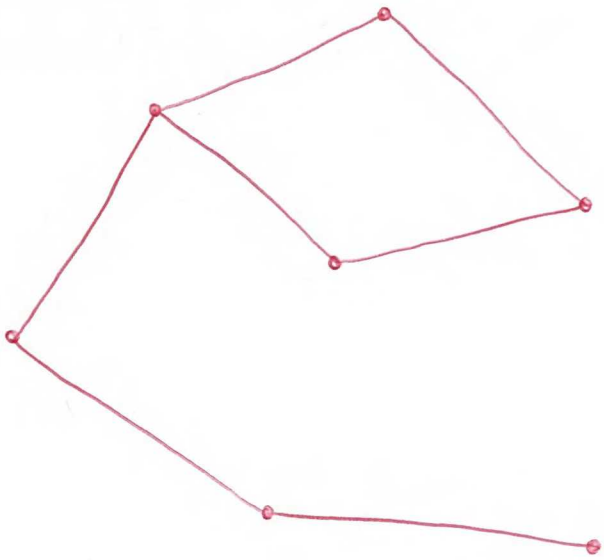
(b) How many edges would a complete graph with 4 vertices have? [2]

$E = \frac{4 \times 3}{2}$ ✓

$E = 6$ ✓

3. (3 marks)

Draw a simple, connected graph with 7 vertices and 7 edges, where one vertex has degree 3 and five vertices have degree 2.



✓ 7V

✓ 7E

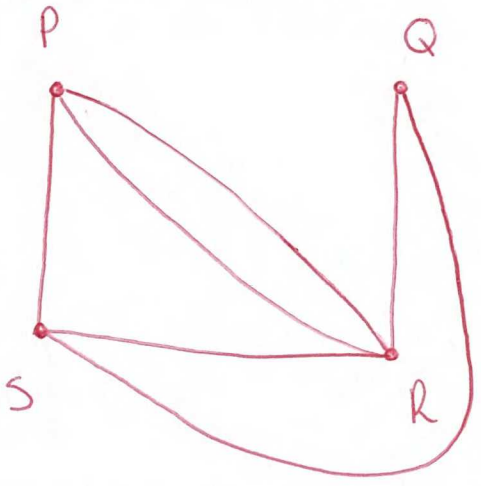
✓ degree's correct

4. (7 marks)

The adjacency matrix, $F$, above represents a planar graph with four vertices.

$$F = \begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(a) Draw the planar graph in the space below. [3]




(b) The number of regions (faces) on the planar graph is: [1]

4

(c) List a circuit which is a subgraph [1]

P - S - Q - R - P

or other correct answer

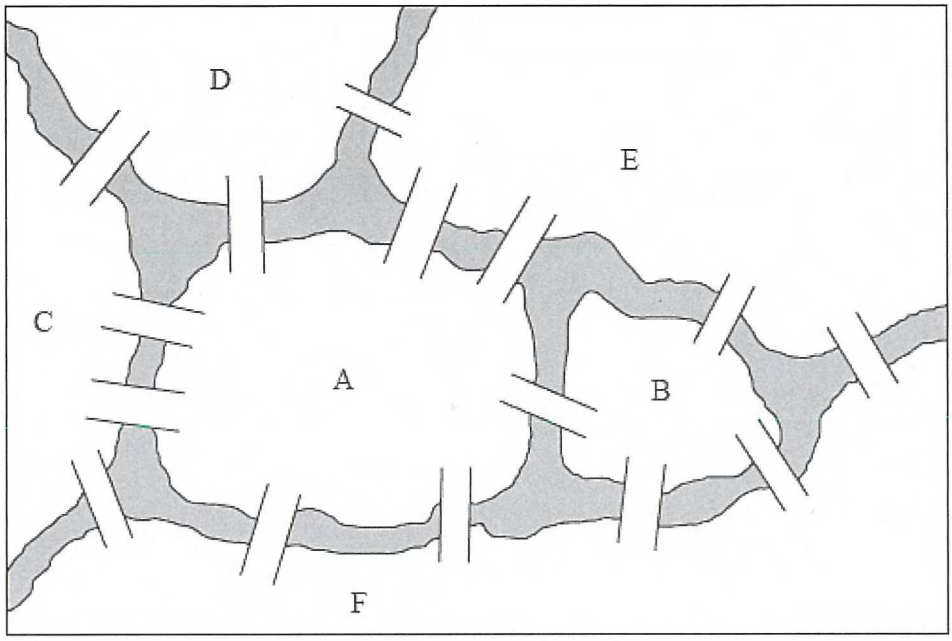
(d) Verify that the network satisfies Euler's rule. [2]

V + F - E = 2

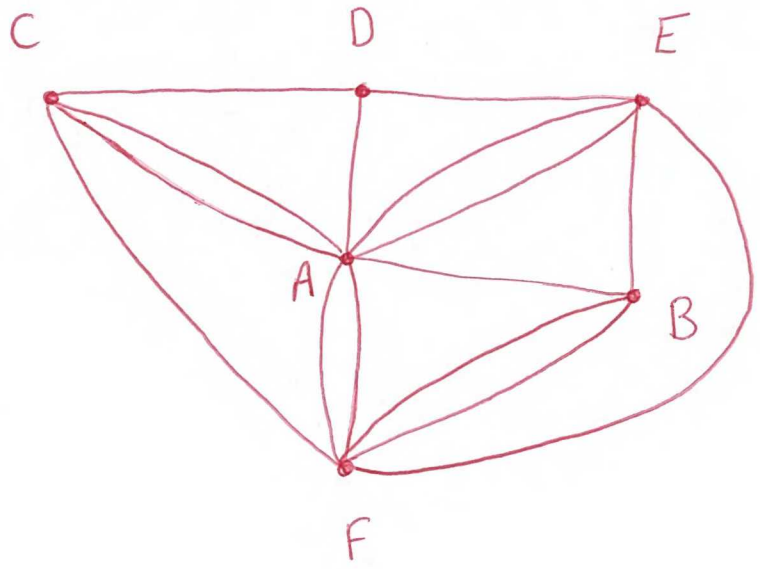
4 + 4 - 6 = 2 ✓✓

5. (8 marks)

A canal system divides a city into six land masses connected by fifteen bridges as shown in the diagram below.




(a) Draw a planar graph to represent this map. [3]




√√√

-1 per error

(b) List the degrees of each of the vertices [2]

| Vertex | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| Degree | 8 | 4 | 4 | 3 | 5 | 6 |

(c) State, with reasons, whether or not this graph has: [2]

(i) an Eulerian circuit

No, all vertices do not have an even degree. Cannot complete every edge once and return to starting point.

(ii) an Eulerian trail.

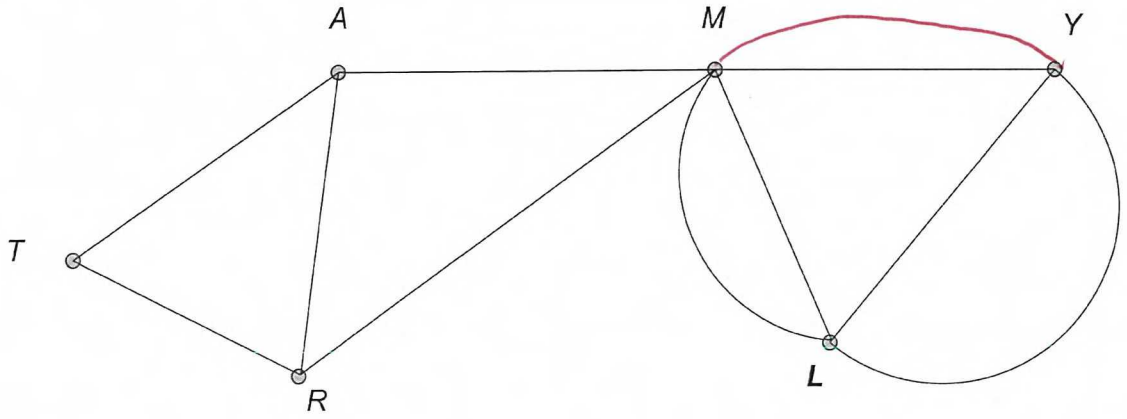
Yes, 2 odd vertices

(d) List a trail of length 4 from E to F [1]

E.g. E - D - C - A - F

6. (8 marks)

A game is played at a school camp. The students have to run over a series of tracks (such as the tracks shown in the diagram below) from one point to another, running over every track exactly once.




(a) Explain why the game is not possible on the set of tracks above. [2]

There are more than 2 odd vertices. Would have to repeat an edge (track).

(b) Draw one more arc on the graph to enable the run [1]

Eg MY or any edge joining 2 odd vertices

(c) State the start and finishing points of the run [1]

For adding MY: start A
Finish R

(d) State the mathematical term used to describe this run [1]

Eulerian Trail

(e) Is the network that represents the original set of tracks a complete graph? Explain. [2]

No, every vertex is not connected to every other vertex.

TRINITY COLLEGE

# Year 12 Mathematics Applications
# Test 2 2019

Section 2 Calculator Assumed

**Graph Theory**

**STUDENT'S NAME:** Solutions

**DATE:** Wednesday 8th May **TIME:** 20 minutes **MARKS:** 18

**INSTRUCTIONS:**

| | |
|---|---|
| Standard Items: | Pens, pencils, drawing templates, eraser |
| Special Items: | Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment) |

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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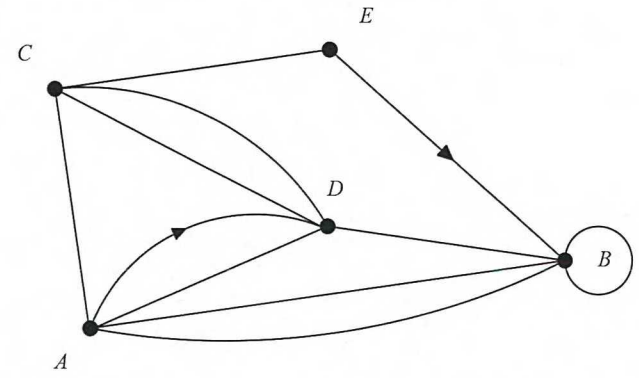
7. (5 marks)

Consider the directed network below.




(a) Construct the adjacency matrix '$M$' that corresponds to this network. [3]

$$M = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 2 & 0 \\ 2 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 & 1 \\ 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

✓✓✓

-1 per error

(b) The matrix $M^3$ is shown below. Explain what this matrix represents. [1]

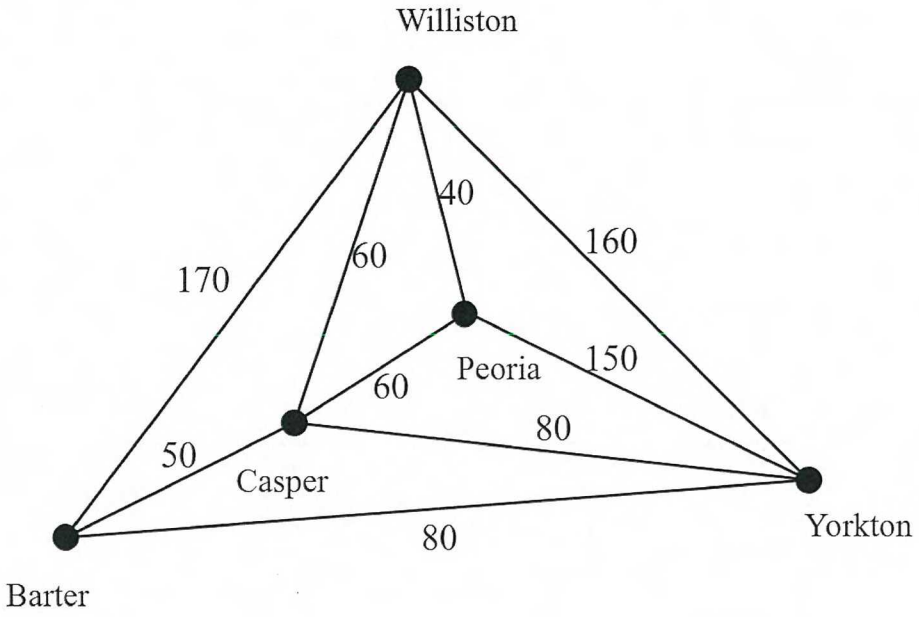
$$M^3 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 20 & 31 & 16 & 28 & 4 \\ 28 & 34 & 17 & 27 & 4 \\ 18 & 16 & 6 & 21 & 6 \\ 16 & 25 & 20 & 14 & 1 \\ 7 & 14 & 10 & 8 & 0 \end{bmatrix} \end{array}$$

Number of 3-stage routes between vertices

8. (4 marks)

The diagram below shows a network of roads between five towns. The numbers indicate the distances, in kilometres, that are travelled between connected towns.

Diagram not to scale




Brodie followed an Eulerian path through this network.

(a) Which towns did he start and finish at? [2]

Barter , Peoria

(b) What distance did he travel? [1]

850 km

Chelsea will follow a Hamiltonian path from Barter to Yorkton.

(c) What is the shortest distance she can travel? [1]

300 km

(B - C - W - P - Y)

9. (10 marks)

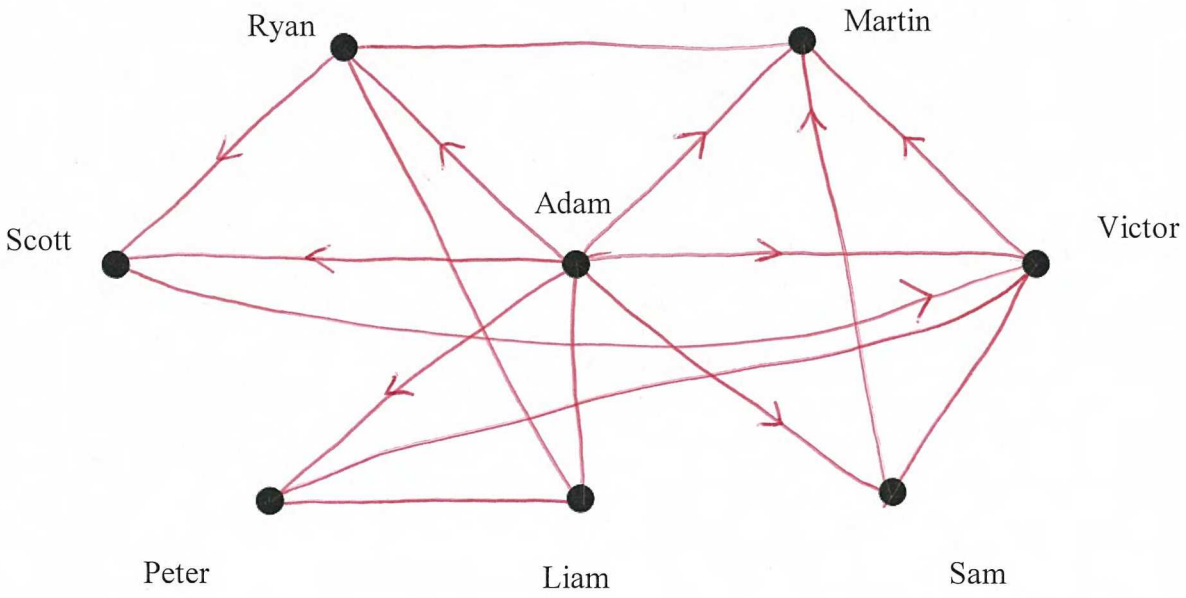
For a group of friends living in close proximity, a line between their names indicates that the friends have both visited each other's house. A directed line indicates that only one friend has visited the house of the other.

For example: Nathan ⟶ Timothy, indicates that Nathan has visited Timothy's house but Timothy has not visited Nathan's house.

In this network:

- Ryan and Martin have visited each other's houses
- Ryan has visited Scott's house
- Scott has visited Victor's house
- Sam has visited Martin's house
- Victor has visited Martin's house
- Victor and Peter have visited each other's houses
- Liam and Peter have visited each other's houses
- Liam and Ryan have visited each other's houses
- Victor and Sam have visited each other's houses
- Adam has visited everyone else's house but only Liam has visited Adam's house.

(a) Complete a directed graph to indicate the above relationships. [3]




√√√

-1 per error

(b) Ryan needs to deliver a book to Sam. If people can only visit houses that they have previously visited, list two different paths that Ryan could use to pass the book on until it is received by Sam. [2]

Ryan - Liam - Adam - Sam

Ryan - Scott - Victor - Sam

(c) Describe the path with the least number of connections between Peter and Scott. [2]

Peter - Liam - Adam - scott

OR

Peter - Liam - Ryan - Scott

(d) Scott is writing a birthday card for another friend (not included in this network) and he would like each of the friends in this network to sign it. Describe the most efficient way of passing the card on to each person's house such that each person is required to make no more than one visit to another house. [3]

Scott → Victor → Sam → Martin → Ryan → Liam

↓

Adam

↓

Peter

✓✓✓

-1 per error

(other answers possible)

OR students can answer with this trail highlighted on the network (re-drawn)

P - V - S - M - R - Sc (6) 5

P - L - R - Scott 3

P - ~~A~~ - A - S

Scott -